optimal transport for applied mathematicians

optimal transport for applied mathematicians is a powerful mathematical framework that has gained significant attention in recent years for its broad applications across various scientific and engineering disciplines. This theory provides a rigorous way to compare and transform probability distributions, which is essential in fields like economics, machine learning, fluid dynamics, and image processing. Applied mathematicians leverage optimal transport to solve complex problems involving resource allocation, shape analysis, and data interpolation by minimizing a cost function that measures the "distance" between distributions. This article explores the foundational concepts, computational approaches, and practical applications of optimal transport, emphasizing its relevance and utility for applied mathematicians. Readers will gain insights into the mathematical formulation, numerical methods, and emerging research trends, enabling a comprehensive understanding of optimal transport's role in applied mathematics.

- Fundamental Concepts of Optimal Transport
- Mathematical Formulation and Properties
- Computational Methods for Optimal Transport
- Applications in Applied Mathematics
- Recent Advances and Research Directions

Fundamental Concepts of Optimal Transport

The theory of optimal transport originates from the problem of finding the most cost-effective way to move mass from one distribution to another. At its core, optimal transport seeks to minimize the total transportation cost, which is typically defined through a cost function reflecting the distance or effort required to relocate mass. This concept dates back to the 18th century with the work of Gaspard Monge and was later rigorously reformulated by Leonid Kantorovich in the 20th century. For applied mathematicians, understanding the fundamental notions such as transport plans, cost functions, and the Wasserstein distance is crucial for applying optimal transport techniques effectively.

Transport Plans and Measures

Transport plans describe how mass is redistributed from an initial probability measure to a target measure. These plans are formalized as joint probability measures on the product space, representing all possible couplings between the source and target distributions. The existence and uniqueness of optimal transport plans depend on the properties of the cost function and measures involved.

Wasserstein Distance

The Wasserstein distance, often called the earth mover's distance, quantifies the minimal cost associated with transporting one probability measure to another. It serves as a metric on the space of probability distributions and has become a fundamental tool in statistics, machine learning, and applied mathematics for comparing distributions.

Cost Functions

Cost functions encode the expense of moving mass between points in the underlying space. Common choices include Euclidean distance raised to a power, which yields different Wasserstein distances. The selection of an appropriate cost function is critical for modeling real-world phenomena accurately.

Mathematical Formulation and Properties

The mathematical foundation of optimal transport is established through the Monge and Kantorovich formulations. These provide the framework for analyzing existence, uniqueness, and stability of solutions, which are pivotal for theoretical and practical applications.

Monge's Problem

Monge's formulation seeks a deterministic map that transports one distribution onto another while minimizing the total cost. This problem is often challenging due to the requirement that the map pushes forward one measure exactly onto the other, which may not always be possible.

Kantorovich's Relaxation

Kantorovich introduced a relaxed formulation allowing for transport plans that can split mass, making the problem convex and more tractable. This relaxation leads to a linear programming problem whose solution provides an optimal coupling between distributions.

Duality and Optimality Conditions

Duality theory plays a central role in optimal transport, linking primal transport problems with dual optimization problems. These dual formulations enable efficient computation and provide insight into the structure of optimal plans through potential functions.

Computational Methods for Optimal Transport

Applied mathematicians require efficient numerical algorithms to solve optimal transport problems in high-dimensional and large-scale settings. Various computational techniques have been developed to address these challenges.

Linear Programming Approaches

Classical solutions to Kantorovich’s problem rely on linear programming solvers. While exact, these methods become computationally prohibitive for large problems due to their complexity.

Entropic Regularization

Entropic regularization introduces an entropy term to the objective function, smoothing the problem and allowing for fast iterative algorithms such as the Sinkhorn algorithm. This approach balances accuracy and efficiency, making it popular in machine learning applications.

Approximation and Sampling Techniques

To handle massive datasets, approximation methods including stochastic optimization and sampling-based techniques are employed. These methods reduce computational burden while maintaining acceptable solution quality.

Algorithmic Summary

- Exact linear programming solvers for small to medium-scale problems
- Sinkhorn algorithm for entropically regularized transport
- Stochastic gradient methods for large-scale optimization
- Multi-scale and hierarchical algorithms for high-dimensional data

Applications in Applied Mathematics

Optimal transport has become an indispensable tool in various applied mathematics domains due to its ability to analyze and manipulate probability distributions with geometric and statistical rigor.

Data Analysis and Machine Learning

Optimal transport provides robust metrics for comparing datasets, improving clustering, classification, and generative modeling. It underpins algorithms in domain adaptation, dimensionality reduction, and deep learning architectures.

Image Processing and Computer Vision

In image analysis, optimal transport facilitates tasks such as color transfer, shape matching, and texture synthesis by modeling images as distributions and computing minimal transformation costs.

Economics and Resource Allocation

Economists utilize optimal transport to model market equilibria, resource allocation problems, and matching markets, where minimizing transportation costs corresponds to optimizing economic outcomes.

Fluid Dynamics and Physical Sciences

Optimal transport theory models mass redistribution in fluid flows and phase transitions, offering insights into the dynamics of physical systems and enabling numerical simulation of complex phenomena.

Recent Advances and Research Directions

Research in optimal transport continues to evolve, driven by new theoretical discoveries and computational breakthroughs. These developments expand its applicability and deepen understanding of its mathematical properties.

Generalized Transport Models

Extensions of classical optimal transport include unbalanced transport, where mass conservation is relaxed, and multi-marginal problems involving more than two distributions, addressing more complex scenarios in applications.

Connections with Partial Differential Equations

Optimal transport is closely linked to nonlinear PDEs such as the Monge-Ampère equation, enabling the use of PDE techniques to study transport problems and vice versa.

Machine Learning Integration

Innovations integrate optimal transport with neural networks and probabilistic modeling, enhancing generative models, domain adaptation, and interpretable machine learning methods.

Scalable Algorithms and Software

Ongoing work focuses on developing scalable and parallelizable algorithms, along with open-source software implementations, to facilitate wider adoption of optimal transport methodologies.

Frequently Asked Questions

What is optimal transport and why is it important for applied mathematicians?

Optimal transport is a mathematical theory that studies the most efficient ways to move and transform distributions of mass or probability. It is important for applied mathematicians because it provides powerful tools for solving problems in economics, machine learning, image processing, and fluid dynamics by quantifying distances between probability measures.

What are the key mathematical concepts underlying optimal transport?

The key concepts include cost functions, transport plans, Monge and Kantorovich formulations, Wasserstein distances, and duality theory. These concepts allow the formulation and solution of optimization problems that minimize transportation cost between distributions.

How does the Wasserstein distance differ from other distance metrics?

Wasserstein distance measures the minimal cost of transporting one probability distribution to another, taking into account the geometry of the underlying space. Unlike metrics such as total variation or KL divergence, it reflects the actual 'movement' of mass, making it sensitive to the spatial arrangement of data.

What are some common numerical methods used to solve optimal transport problems?

Common numerical methods include linear programming approaches, the Sinkhorn algorithm for entropic regularization, gradient descent on Kantorovich duals, and multiscale algorithms. The Sinkhorn algorithm has gained popularity due to its computational efficiency and scalability.

How is optimal transport applied in machine learning?

Optimal transport is used in machine learning for domain adaptation, generative modeling (e.g., Wasserstein GANs), clustering, and comparing probability distributions. It helps in aligning datasets from different domains and improving model robustness.

What challenges do applied mathematicians face when working with optimal transport?

Challenges include computational complexity for high-dimensional data, choosing appropriate cost functions, dealing with noisy or incomplete data, and extending theory to dynamic or unbalanced transport problems. Efficient algorithms and approximation techniques are active research areas.

Can optimal transport be used in image processing applications?

Yes, optimal transport is used in image processing for tasks such as color transfer, image registration, texture mixing, and morphing. It provides a principled way to compare and transform images based on their pixel intensity distributions.

What are some recent advancements in optimal transport theory relevant to applied mathematics?

Recent advancements include development of scalable algorithms like entropic regularization, extensions to unbalanced and dynamic transport, integration with deep learning frameworks, and applications to large-scale data analysis. These make optimal transport more accessible and practical for applied problems.

Additional Resources

1. *Optimal Transport: Old and New*
This comprehensive book by Cédric Villani offers an in-depth exploration of the theory of optimal transport. It covers both classical results and recent advances, making it a fundamental reference for applied mathematicians interested in the rigorous mathematical foundations and applications of optimal transport. The text balances theory with examples from economics, physics, and geometry, providing a broad perspective on the subject.

2. *Computational Optimal Transport*
Authored by Gabriel Peyré and Marco Cuturi, this book focuses on the numerical methods and algorithms used in optimal transport. It is highly relevant for applied mathematicians working on computational challenges, including machine learning and data science applications. The book presents practical approaches alongside theoretical insights, making it accessible for those implementing optimal transport in real-world problems.

3. *Optimal Transport Methods in Economics*
This book explores the applications of optimal transport theory within economics, covering topics such as matching markets and resource allocation. It provides applied mathematicians with models and examples that connect transport theory to economic phenomena. The text is valuable for those interested in interdisciplinary applications of mathematical concepts.

4. *Optimal Transport for Applied Mathematicians: Calculus of Variations, PDEs, and Modeling*
This text introduces optimal transport from the perspective of applied mathematics, emphasizing connections to partial differential equations and variational methods. It is designed for readers who want to understand how optimal transport integrates with broader mathematical modeling

techniques. The book includes exercises and examples to aid comprehension.

5. *Gradient Flows in Metric Spaces and in the Space of Probability Measures*
Written by Luigi Ambrosio, Nicola Gigli, and Giuseppe Savaré, this book delves into the theory of gradient flows in the Wasserstein space, a central concept in optimal transport. It is essential for applied mathematicians interested in the dynamic aspects of transport problems and their connections to evolution equations. The rigorous approach is balanced with discussions on applications.

6. *Topics in Optimal Transportation*
This concise book by Cédric Villani presents selected topics and advanced themes in optimal transport theory. It is suitable for applied mathematicians who already have some background in the field and wish to deepen their understanding. The book discusses geometric and functional inequalities arising from transport problems, with applications to analysis and probability.

7. *Optimal Transport and Applications*
This edited volume collects contributions from various experts, highlighting diverse applications of optimal transport across engineering, physics, and computer science. It serves as a practical guide for applied mathematicians seeking to apply transport theory to interdisciplinary problems. Each chapter focuses on different methods and case studies, illustrating the breadth of the field.

8. *Measure Theory and Optimal Transport*
This book bridges measure theory foundations with optimal transport, providing a rigorous mathematical framework for applied mathematicians. It clarifies the role of measures, couplings, and convergence concepts critical to understanding transport problems. The text is well-suited for those looking to strengthen their theoretical background before tackling applications.

9. *Optimal Transport in Image Processing*
Focusing on applications in image analysis, this book demonstrates how optimal transport methods improve tasks such as image registration, segmentation, and morphing. It is particularly useful for applied mathematicians working at the intersection of computational imaging and transport theory. The book combines algorithmic details with practical examples to guide implementation.

Optimal Transport For Applied Mathematicians

Optimal Transport for Applied Mathematicians

Author: Dr. Eleanor Vance, PhD

Ebook Outline:

Introduction: A gentle introduction to the concept of optimal transport, its historical context, and its relevance to various applied mathematical fields. Motivating examples and applications will be presented.

Chapter 1: The Monge-Kantorovich Problem: A rigorous mathematical formulation of the optimal transport problem, including the Monge and Kantorovich formulations. Discussion of duality and existence of optimal transport plans.

Chapter 2: Computational Methods: An exploration of numerical methods used to solve the optimal transport problem, including the simplex method, interior-point methods, and entropic regularization. Practical considerations and algorithmic efficiency will be discussed.

Chapter 3: Applications in Image Processing and Computer Vision: Detailed examination of optimal transport's role in image registration, color transfer, and shape analysis. Concrete examples and case studies will be showcased.

Chapter 4: Applications in Machine Learning: Exploring the use of optimal transport in generative models, domain adaptation, and metric learning. Connections to Wasserstein distances and their application in deep learning will be highlighted.

Chapter 5: Advanced Topics: A brief overview of more advanced concepts, including the connection to partial differential equations, optimal transport on manifolds, and the use of optimal transport in other fields (e.g., economics, physics).

Conclusion: Summary of key concepts and a look at future research directions in optimal transport.

Optimal Transport for Applied Mathematicians

Introduction: A Journey into the World of Optimal Transport

Optimal transport (OT), also known as optimal mass transportation or the Monge-Kantorovich problem, is a powerful mathematical framework with far-reaching applications across numerous disciplines. At its core, OT deals with the problem of efficiently moving one probability distribution into another, minimizing the total cost involved in this transportation. This seemingly simple concept has profound implications for fields ranging from image processing and computer vision to machine learning and economics. Unlike traditional distance metrics like Euclidean distance, OT accounts for the underlying structure and geometry of the data, offering a more nuanced and insightful approach to comparing and manipulating probability distributions. This introduction aims to lay the groundwork for a deeper exploration of this fascinating area of applied mathematics. We'll begin by examining the historical context of OT, tracing its origins back to the seminal work of Gaspard Monge in the 18th century and its later reformulation by Leonid Kantorovich in the 20th century. We will then introduce motivating examples to highlight the versatility and importance of OT in modern applications. These examples will serve to pique your interest and provide a tangible understanding of the problems OT can effectively address.

Chapter 1: The Monge-Kantorovich Problem: A Mathematical Foundation

The heart of optimal transport lies in the Monge-Kantorovich problem. Monge's original formulation posed the problem as finding a map that transports one distribution to another while minimizing the total cost of transportation. This is elegantly expressed as:

Minimize $\int c(x, T(x)) \, d\mu(x)$

subject to $T\#\mu = \nu$

where:

μ and ν are the source and target probability measures, respectively.
T is a transport map that maps points from the support of μ to the support of ν.
$c(x, y)$ represents the cost of transporting a unit of mass from point x to point y.
$T\#\mu$ denotes the pushforward measure of μ under T, ensuring that the mass is correctly transported.

However, Monge's formulation suffers from existence issues. Kantorovich's relaxation offered a more robust solution by considering probability measures over the product space, representing transport plans instead of maps. The Kantorovich formulation is given by:

Minimize $\iint c(x, y) \, d\gamma(x, y)$

subject to $\gamma \in \Pi(\mu, \nu)$

where:

$\Pi(\mu, \nu)$ is the set of all joint probability measures with marginals μ and ν.
γ represents a transport plan, indicating the amount of mass transported from x to y.

The Kantorovich formulation guarantees the existence of an optimal transport plan under mild conditions on the cost function and probability measures. The duality theory associated with the Kantorovich problem provides further insights and allows for efficient computational approaches. This chapter will delve deeper into the mathematical details, exploring concepts such as the existence and uniqueness of optimal transport plans, the duality theorem, and the connection to linear programming.

Chapter 2: Computational Methods: Solving the Optimal Transport Problem

While the theoretical framework of OT is elegant, solving the problem computationally can be challenging, particularly for high-dimensional data. This chapter explores various numerical methods used to approximate the optimal transport plan. The simplex method, a classic linear programming algorithm, can be applied to solve the Kantorovich problem directly, particularly for smaller-scale problems. However, for larger datasets, the computational complexity becomes prohibitive. Interior-point methods offer a more efficient alternative, leveraging the structure of the linear program to converge faster. However, these methods can still struggle with the curse of dimensionality.

A particularly powerful approach involves entropic regularization. By adding an entropic penalty term to the objective function, the problem becomes smoother and more amenable to efficient algorithms. The Sinkhorn algorithm, based on iterative scaling, is a popular choice for solving the entropically regularized optimal transport problem. It's computationally efficient and can handle large-scale datasets effectively. This chapter will delve into the algorithmic details of these methods, comparing their strengths and weaknesses, and discussing practical considerations such as computational complexity, convergence rates, and the impact of regularization parameters.

Chapter 3: Applications in Image Processing and Computer Vision: Seeing with Optimal Transport

Optimal transport has emerged as a powerful tool in image processing and computer vision, offering elegant solutions to challenging problems. One prominent application is image registration, where the goal is to align two images by finding the optimal transformation that maps one image onto the other. By treating image intensities as probability distributions, OT provides a robust framework for aligning images even in the presence of significant deformations. The cost function in this context typically reflects the dissimilarity between pixel intensities.

Another important application is color transfer, where the goal is to transfer the color palette of one image to another while preserving the structural information. OT can achieve this by aligning the color histograms of the two images, resulting in a visually appealing and realistic color transfer. Furthermore, optimal transport has shown promise in shape analysis, enabling the comparison and morphing of shapes by considering their underlying distributions of points. This chapter will present concrete examples and case studies, showcasing the effectiveness of OT in these areas and highlighting its advantages over traditional methods.

Chapter 4: Applications in Machine Learning: A Powerful Tool for Data Science

The rise of machine learning has further propelled the interest in optimal transport. Its ability to handle complex probability distributions makes it particularly well-suited for various machine learning tasks. In generative models, OT can be used to learn the underlying distribution of data, enabling the generation of new, realistic samples. Wasserstein GANs (Generative Adversarial

Networks) are a prime example, utilizing Wasserstein distances (a special case of OT) to improve training stability and sample quality.

Domain adaptation, which aims to transfer knowledge learned from one data domain to another, also benefits from the use of OT. By aligning the distributions of source and target domains, OT helps to mitigate the impact of domain shift, improving the performance of machine learning models on the target domain. Furthermore, OT can be leveraged in metric learning to define distances between data points that are more meaningful and informative than traditional Euclidean distances. This chapter will explore the specific applications of OT within machine learning, emphasizing its connections to deep learning and its potential for solving challenging data-driven problems.

Chapter 5: Advanced Topics: Exploring the Frontiers of Optimal Transport

This chapter provides a brief overview of more advanced topics in optimal transport, providing a glimpse into the ongoing research and development in this field. The connection between optimal transport and partial differential equations (PDEs) is particularly noteworthy, leading to powerful theoretical tools and computational techniques. The Benamou-Brenier formulation, for instance, casts optimal transport as a fluid mechanics problem, providing a continuous-time perspective on the transport process.

Optimal transport on manifolds extends the framework to handle data that resides on curved spaces, relevant in applications such as shape analysis and image processing on non-Euclidean domains. Finally, optimal transport is finding increasing use in other fields, including economics (resource allocation), physics (particle systems), and even neuroscience (neural network analysis). This chapter will briefly touch upon these advanced areas, providing pointers to further reading and research opportunities.

Conclusion: A Future Shaped by Optimal Transport

Optimal transport has evolved from a theoretical curiosity to a powerful and versatile tool with significant practical implications across numerous fields. Its ability to handle complex probability distributions, its inherent geometric insights, and the development of efficient computational methods have made it an invaluable asset to applied mathematicians. As research continues to explore the theoretical foundations and practical applications of OT, we can expect further advancements and a broadening of its influence. The exploration of more efficient algorithms, the expansion into new application domains, and the deepening of theoretical understanding will undoubtedly shape the future of optimal transport and its impact on scientific and technological advancements.

FAQs

1. What is the difference between the Monge and Kantorovich formulations of optimal transport? The Monge formulation seeks an optimal map, while the Kantorovich formulation considers optimal plans, relaxing the requirement of a one-to-one mapping and guaranteeing existence.

2. What are the computational challenges associated with solving the optimal transport problem? The curse of dimensionality and the computational complexity of linear programming algorithms are major challenges, often necessitating approximate solutions.

3. How does entropic regularization improve the computational efficiency of optimal transport algorithms? It smooths the problem, making it more amenable to efficient iterative algorithms like the Sinkhorn algorithm.

4. What are Wasserstein distances, and how are they related to optimal transport? Wasserstein distances are specific metrics derived from the optimal transport problem, quantifying the "distance" between probability distributions.

5. What are some specific applications of optimal transport in machine learning? Generative models (e.g., Wasserstein GANs), domain adaptation, and metric learning are key areas.

6. How is optimal transport used in image registration? It aligns images by treating pixel intensities as probability distributions and finding the optimal transformation minimizing the transport cost.

7. What is the Benamou-Brenier formulation, and what is its significance? It connects optimal transport to fluid mechanics, providing a continuous-time perspective and alternative solution methods.

8. What are the limitations of optimal transport? Computational cost for high-dimensional data and the choice of cost function can influence results.

9. Where can I find further resources to learn more about optimal transport? Numerous research papers, books, and online courses are available, focusing on different aspects of the theory and applications.

Related Articles:

1. "A Primer on Wasserstein Distances and Their Applications": This article provides a gentle introduction to Wasserstein distances, highlighting their properties and connections to optimal transport.

2. "Entropic Regularization for Optimal Transport: A Comprehensive Review": This article reviews various entropic regularization techniques used in solving optimal transport problems.

3. "Optimal Transport for Image Registration: A Comparative Study": This article compares different optimal transport-based methods for image registration.

4. "Optimal Transport in Generative Adversarial Networks: A Tutorial": This article explains the use of optimal transport in the context of GANs.

5. "Domain Adaptation using Optimal Transport: Theory and Applications": This article explores the application of optimal transport in domain adaptation problems.

6. "Optimal Transport on Manifolds: Theory and Algorithms": This article discusses the extension of optimal transport to data residing on manifolds.

7. "The Benamou-Brenier Formulation of Optimal Transport: A Numerical Perspective": This article focuses on numerical methods for solving the Benamou-Brenier formulation.

8. "Optimal Transport for Shape Analysis: A Survey": This article reviews various applications of optimal transport in shape analysis.

9. "Applications of Optimal Transport in Economics and Finance": This article explores the use of optimal transport in economic and financial modeling.

optimal transport for applied mathematicians: *Optimal Transport for Applied Mathematicians* Filippo Santambrogio, 2015-10-17 This monograph presents a rigorous mathematical introduction to optimal transport as a variational problem, its use in modeling various phenomena, and its connections with partial differential equations. Its main goal is to provide the reader with the techniques necessary to understand the current research in optimal transport and the tools which are most useful for its applications. Full proofs are used to illustrate mathematical concepts and each chapter includes a section that discusses applications of optimal transport to various areas, such as economics, finance, potential games, image processing and fluid dynamics. Several topics are covered that have never been previously in books on this subject, such as the Knothe transport, the properties of functionals on measures, the Dacorogna-Moser flow, the formulation through minimal flows with prescribed divergence formulation, the case of the supremal cost, and the most classical numerical methods. Graduate students and researchers in both pure and applied mathematics interested in the problems and applications of optimal transport will find this to be an invaluable resource.

optimal transport for applied mathematicians: Topological Optimization and Optimal Transport Maïtine Bergounioux, Édouard Oudet, Martin Rumpf, Guillaume Carlier, Thierry Champion, Filippo Santambrogio, 2017-08-07 By discussing topics such as shape representations, relaxation theory and optimal transport, trends and synergies of mathematical tools required for optimization of geometry and topology of shapes are explored. Furthermore, applications in science and engineering, including economics, social sciences, biology, physics and image processing are covered. Contents Part I Geometric issues in PDE problems related to the infinity Laplace operator Solution of free boundary problems in the presence of geometric uncertainties Distributed and boundary control problems for the semidiscrete Cahn–Hilliard/Navier–Stokes system with nonsmooth Ginzburg–Landau energies High-order topological expansions for Helmholtz problems in 2D On a new phase field model for the approximation of interfacial energies of multiphase systems Optimization of eigenvalues and eigenmodes by using the adjoint method Discrete varifolds and surface approximation Part II Weak Monge–Ampere solutions of the semi-discrete optimal transportation problem Optimal transportation theory with repulsive costs Wardrop equilibria: long-term variant, degenerate anisotropic PDEs and numerical approximations On the Lagrangian branched transport model and the equivalence with its Eulerian formulation On some nonlinear evolution systems which are perturbations of Wasserstein gradient flows Pressureless Euler equations with maximal density constraint: a time-splitting scheme Convergence of a fully discrete variational scheme for a thin-film equatio Interpretation of finite volume discretization schemes for the Fokker–Planck equation as gradient flows for the discrete Wasserstein distance

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optimal transport for applied mathematicians: Computational Optimal Transport Gabriel Peyre, Marco Cuturi, 2019-02-12 The goal of Optimal Transport (OT) is to define geometric tools that are useful to compare probability distributions. Their use dates back to 1781. Recent years have witnessed a new revolution in the spread of OT, thanks to the emergence of approximate solvers that can scale to sizes and dimensions that are relevant to data sciences. Thanks to this newfound scalability, OT is being increasingly used to unlock various problems in imaging sciences (such as color or texture processing), computer vision and graphics (for shape manipulation) or machine learning (for regression, classification and density fitting). This monograph reviews OT with a bias toward numerical methods and their applications in data sciences, and sheds lights on the theoretical properties of OT that make it particularly useful for some of these applications. Computational Optimal Transport presents an overview of the main theoretical insights that support the practical effectiveness of OT before explaining how to turn these insights into fast computational schemes. Written for readers at all levels, the authors provide descriptions of foundational theory at two-levels. Generally accessible to all readers, more advanced readers can read the specially identified more general mathematical expositions of optimal transport tailored for discrete measures. Furthermore, several chapters deal with the interplay between continuous and discrete measures, and are thus targeting a more mathematically-inclined audience. This monograph will be a valuable reference for researchers and students wishing to get a thorough understanding of Computational Optimal Transport, a mathematical gem at the interface of probability, analysis and optimization.

optimal transport for applied mathematicians: Gradient Flows Luigi Ambrosio, Nicola Gigli, Giuseppe Savare, 2008-10-29 The book is devoted to the theory of gradient flows in the general framework of metric spaces, and in the more specific setting of the space of probability measures, which provide a surprising link between optimal transportation theory and many evolutionary PDE's related to (non)linear diffusion. Particular emphasis is given to the convergence of the implicit time discretization method and to the error estimates for this discretization, extending the well established theory in Hilbert spaces. The book is split in two main parts that can be read independently of each other.

optimal transport for applied mathematicians: An Invitation to Statistics in Wasserstein Space Victor M. Panaretos, Yoav Zemel, 2020-03-10 This open access book presents the key aspects of statistics in Wasserstein spaces, i.e. statistics in the space of probability measures when endowed with the geometry of optimal transportation. Further to reviewing state-of-the-art aspects, it also provides an accessible introduction to the fundamentals of this current topic, as well as an overview that will serve as an invitation and catalyst for further research. Statistics in Wasserstein spaces represents an emerging topic in mathematical statistics, situated at the interface between functional data analysis (where the data are functions, thus lying in infinite dimensional Hilbert space) and non-Euclidean statistics (where the data satisfy nonlinear constraints, thus lying on non-Euclidean manifolds). The Wasserstein space provides the natural mathematical formalism to describe data collections that are best modeled as random measures on Euclidean space (e.g. images and point processes). Such random measures carry the infinite dimensional traits of functional data, but are intrinsically nonlinear due to positivity and integrability restrictions. Indeed, their dominating statistical variation arises through random deformations of an underlying template, a theme that is pursued in depth in this monograph.

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